Performance Prediction Analysis for Fully Deployed Parachute Canopies

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In this paper, by introducing experimentally determined apparent mass terms into the equations of motion, the performance characteristics of the descending parachute store system have been determined, and they resemble those observed enough to be an appropriate basis for performance prediction. Using phase lag instead of the variable apparent mass term in the equations, the dynamic stability of the system has also been analyzed. It was found that, for a parachute with a high value of $dC_N/d\alpha$ about the equilibrium angle, the influences of the apparent masses and the phase lag on the performance characteristics of the system are not significant.

Nomenclature

A	$= \pi D^2/4$; cross-sectional area of parachute canopy
D_D	= drag force coefficient
D	= nominal diameter
$F(x, y, z)$	= external force
$M(L, M, N)$	= external moment
I_f	$= \pi\rho D^2 \forall/16$; representative moment of inertia of the fluid displaced by the body
$V(u, v, w)$	= velocity components along x, y, z axes
$\omega(p, q, r)$	= angular velocity components about x, y, z axes
V_0	= equilibrium velocity
$\forall$	$= \pi D^3/12$; representative volume of fluid displaced by parachute
m	= mass of parachute and store
L	= parachute rigging line length
R_0	= center of reaction of parachute canopy
C_N	$= N/\frac{1}{2}\rho V_0^2 A$; normal force coefficient
C_T	$= T/\frac{1}{2}\rho V_0^2 A$; tangential force coefficient
C_{N_α}	$= dC_N/d\alpha$; normal force coefficient slope
α_{11}	= apparent mass component in ox direction
α_{33}	= apparent mass component in oz direction
α_{55}	= apparent moment of inertia component
k_{ij}	$= \alpha_{ij}/\rho\forall$; generalized apparent mass coefficient, $i,j = 1$–3
k_{ij}	$= \alpha_{ij}/I_f$; generalized apparent moment coefficient, $i,j = 4$–6
k_{11}, k_{22}, k_{33}	= apparent mass coefficients along ox, oy, oz axes
$k_{44}, k_{55}\ k_{66}$	= apparent moment of inertia coefficients about ox, oy, oz axes
α	= angle of attack
δ	$= \dot{V}D/V^2$; acceleration modulus
λ	$= \lambda \mp i\omega$; complex eigenvalues of the canopy and store pitching oscillation
ω	= frequency of oscillation
τ	= time lag
θ	= pitch angle
ρ	= density

Received March 23, 1987; revision received April 15, 1988.

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I. Introduction

ALTHOUGH parachutes have been in service for at least 60 years, their aerodynamic design and surrounding flow phenomenon are still unknown. It is very difficult to determine what specific changes of canopy shape would be required to increase the damping rate of its motion and to reduce its oscillation by a specified frequency or to affect its drive velocity.

Whatever its function, the pitching motion of a parachute during its descent is a mater of extreme importance. Because of the extreme initial and environmental conditions under which parachutes may have to operate, it can be difficult to observe and measure the dynamic response of a fully deployed parachute canopy during its motion. The equations of motion that describe the behavior of a parachute canopy rigidly coupled to the store necessarily include the two significant apparent mass components α_{11} and α_{33}, and for a given parachute canopy descending through real fluid, these must be determined experimentally. The magnitudes of these components are altitude- and acceleration- (or time-) dependent.[1]

Because of the complexity of the experimental expression for the apparent mass components, later experiments conducted by Leicester University Parachute Research Group aimed at a revised and possibly more profitable approach, the use of the stability derivatives. The so-called Polpitye model[2] uses the force balance equation

$$F(t) = \tfrac{1}{2}\rho C_D A V(t-\tau)|V(t-\tau)| + k\rho\forall\dot{V}(t) \qquad (1)$$

where τ is the time lag, which might be constant for the oscillatory motion of a given parachute canopy. In this equation, there are three unknowns to specify: C_D, k, and τ. Polpitye takes C_D to be equal to its steady-state value and k to be the potential flow estimate, leaving one free parameter τ, which he determined from experimental data. Thus, it might permit the adoption of constant rather than variable parameters α_{11} and α_{33} in the equations of motion. At present, it is not known whether the time lag is dependent on the amplitude and frequency of oscillation or the bluff-body shape and size.

Introducing experimentally determined apparent mass terms,[1] which are functions of both δ and α, into the equations of motion, the performance characteristics of parachutes were determined. The effects of pitching oscillation caused by certain changes in system physical parameters, namely, the size of the canopy, the lengths of rigging lines, and the store mass, have been considered as well as the effect of parachute altitude.

II. Nature of the Total Force in Unsteady Motion of Parchute Store System

Among those who have played a notable part in deriving mathematical models for a descending parachute are Henn,[3] who considered planer motion and included the virtual inertia of the air; Lester,[4] who showed that, in general, in the equations of motion four apparent inertia coefficients are significant; Wolf[5] and White and Wolf,[6] who represented the parachute and its store as a two-body problem; Eaton,[7] who considered three apparent mass terms in the equations and concluded that apparent mass effects are stronger than previously predicted; and Doherr and Saliaris,[8] who analyzed the influence of fluctuating aerodynamic forces and of varying inertia coefficients on the dynamic stability of parachutes. Following very early experimental work conducted by Frazer and Simmons[9] and Relf and Jones,[10] Yavuz and Cockrell[1] measured apparent mass components for model parachutes on which descent motion was represented by simultaneously translational and oscillatory modes. Using these experimental values, Yavuz[11,12] determined the performance prediction of fully deployed parachute descent.

The apparent mass terms within the equations of motion for a parachute store system are the means of quantifying the changing fluid motion occurring as a consequence of a parachute disturbance. If potential flow is assumed, Lamb[13] and Ibrahim[14] have shown how such terms may be evaluated.

Defining the apparent mass components for parachute canopies[12] and using the symbols in Fig. 1, for a motion restricted to be within one plane of symmetry, the generally accepted expressions for the total external aerodynamic force components X, Z and external pitching moment M are

$$X = (m + \alpha_{11})\,\dot{u} + (m + \alpha_{33})\,qw + (mz_s + \alpha_{15})\,q$$

$$Z = (m + \alpha_{33})\,\dot{w} + (m + \alpha_{11})\,qu + (mz_s + \alpha_{15})\,q^2$$

$$M = (I_{yy} + \alpha_{55} + mz_s^2 + \alpha_{11}\,z_a^2)\,\dot{q} + (mz_s + \alpha_{15})$$

$$(\dot{u} + qw) + (\alpha_{11} - \alpha_{33})\,uw \qquad (2)$$

In these equations u, w, and q denote components of linear and angular velocities along the axes ox and oz and about the axis oy, respectively. The axes are fixed in the parachute system, as shown in Fig. 1. The mass of the system is m, and its centroid is on the z axis at the distance z_s from origin 0. Apparent mass terms are denoted by α_{11}, α_{33}, α_{15}, and α_{55}, where subscripts 1, 2, 3, 4, and 5 represent motion along or about the axis ox, oy, and oz, respectively. The symbol α_{15} is the apparent inertia compotent in the y direction resulting from acceleration in the x direction, and $\alpha_{15} = \alpha_{51} = \alpha_{11} \cdot z_a$. If the coordinate system origin were to be coincident with the center of pressure at which apparent inertia terms were defined, then Yavuz[15] shows that the apparent inertia component α_{15} is zero.

Equations (2) have been formed by analysis of a freely moving body through a potential (i.e., incompressible and inviscid) flowfield, and although experimental values of the apparent mass components (in Fig. 2) were being inserted into the parachute prediction computer model, the general validity of these equations is open to question.

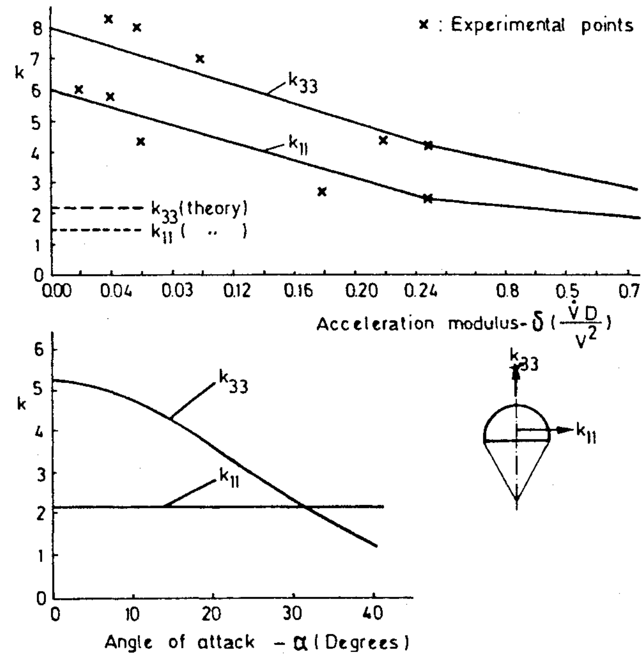

Fig. 2a Variations of apparent mass coefficients k_{11} and k_{33} with acceleration modulus δ and angle of attack α.[1]

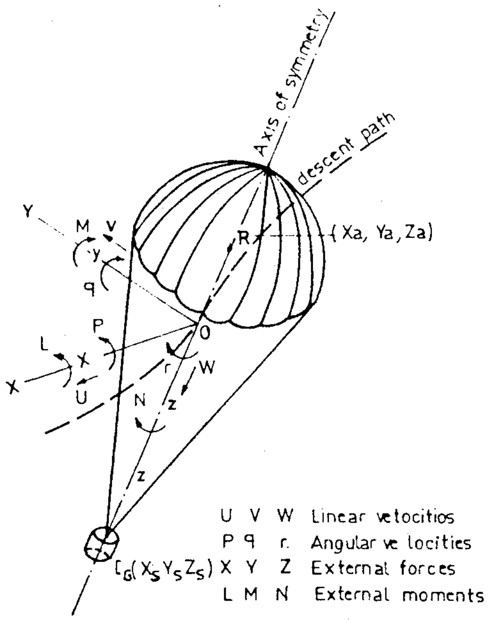

Fig. 1 System axes and sign convention adopted for conventional parachutes.

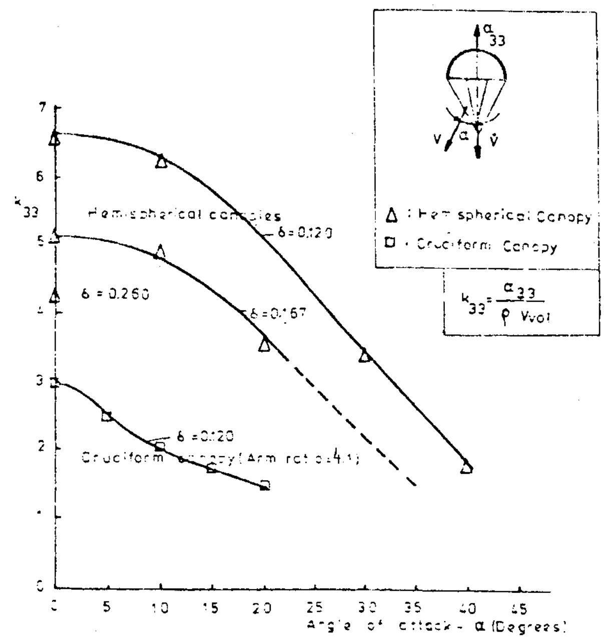

Fig. 2b Variation of k_{33} with angle of attack α for hemispherical and cruciform canopies.

Tests conducted on a rigid hemispherical canopy model by the Leicester Parachute Aerodynamic Research Group[16] provided the necessary information about the validity of the equations. The model immersed in water was oscillated with a varying angular velocity q while translating axially along the ship tank with a constant velocity U_o. When test data were used to determine α_{11} and α_{33} from Eqs. (2), it was discovered that the forces X and Z, which resulted from the acceleration of the system $\dot{u}$ and $\dot{w}$, were not in phase with the corresponding accelerations. The reasons for this imbalance are discussed in detail in Ref. 17, the most important being the phase lag difference.

Phase lag is a characteristic of unsteady bluff-body motion through fluids. It has been modeled for ship maneuvers by Bishop et al.,[18] who refer to it as a memory effect. Similarly, Chan et al.[19] detected a phase lag between the periodic motion of a bluff-body model in air (a cargo container slung beneath a helicopter) and the pitching moment that caused this motion. Flow visualization studies were conducted by Verley and Moe[20] on periodically oscillating circular cylinders immersed in water. In these, they show that at a given instantaneous velocity, the point at which the flow separates behind oscillating bluff bodies may be different from those at which it would separate in steady motion at the same velocity. Since the positions of flow separation play a significant role in determining velocity-dependent drag, the quasisteady assumptions in Eqs. (2) may well be invalid. There is a phase difference of about 90 deg between the canopy periodic velocity and its periodic acceleration. Thus the phase lag of supposed acceleration-dependent forces on the periodic acceleration may be atrributed to the presence of some velocity-dependent contribution, either positive or negative. Phase difference can be observed between the total forces N and T and their associated periodic accelerations $\dot{u}$ and $\dot{w}$ for canopies, as shown in Fig. 3. They were also evident in the translational acceleration experiments devised to establish values of α_{11} and α_{22}.

From experimental work, Polpitye[2] has found that the time lag is approximately 0.3 s for a hemispherical canopy having a 5-s oscillatory period. At present, it is not known if this phase lag (i.e., time lag) is dependent on the characteristics of motion. Therefore, better experimental measurements of unsteady aerodynamic forces on parachute canopies are needed.

III. Linearized Stability Analysis

From Eqs. (2), linearized stability analysis for a parachute store system can be developed.[15] The aerodynamic force produced by store is neglected and that on the canopy written in terms of normal and axial force coefficients C_N and C_T, which are assumed to be known functions of the instantaneous angle of attack. For a canopy that is initially in equilibrium at zero angle of attack, using small-disturbance theory, the characteristic polynomial of the eigenvalues $\lambda = \lambda + i\omega$ for the system can be expressed[15] as

$$K(\lambda) = A_3\,\lambda^3 + A_2\,\lambda^2 + A_1\lambda + A_0 \tag{3}$$

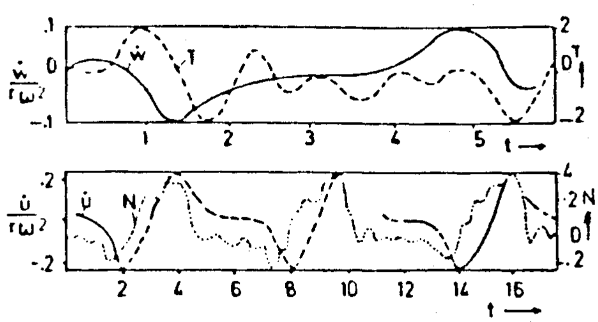

Fig. 3 Phase difference between forces (N and T) and kinematic variables ($\dot{u}$, $\dot{w}$).[16]

in which

$$A_3 = a_1 d_2 - c_1 b_2,\quad A_2 = a_1 d_1 + d_0 d_2 - c_0 b_2 - c_1 b_1$$

$$A_1 = a_1 d_0 + a_0 d_1 - c_1 b_0 - c_0 b_1$$

$$A_0 = a_0 d_0 - c_0 b_0$$

$$a_1 = \frac{m + a_{11}}{m} \qquad c_1 = \frac{m z_s + \alpha_{11} z_a}{mL}$$

$$a_0 = \frac{gL}{C_T V_0^2} \qquad c_0 = \frac{g z_a}{V_0^2\, C_T}\frac{dC_N}{d\alpha} + \frac{\alpha_{11}\, z_a}{L}$$

$$b_2 = \frac{m z_s}{m} + \frac{gL}{C_T V_0^2} \qquad d_2 = \frac{I_{yy} + m z_s^2 + a_{55} + a_{11}\, z_a^2}{mL^2}$$

$$b_1 = \frac{m + \alpha_{11}}{m} + \frac{g z_a}{V_0^2\, C_T}\frac{dC_N}{d\alpha}$$

$$d_1 = \frac{g z_a^2}{V_0^2\, L C_T}\frac{dC_N}{d\alpha} + \frac{m z_s + \alpha_{11} z_a}{mL}$$

$$b_0 = \frac{gL}{V_0^2} \qquad d_0 = \frac{g z_s}{V_0^2}$$

Using Routh's stability criterion, it can be simply demonstrated that the static stability condition

$$C_{N_\alpha} = \frac{dC_N}{d\alpha} > 0 \tag{4}$$

is one that must be satisfied if dynamic stability is to be achieved. Figures 4 and 5 illustrate that the static stability criterion is a necessary but insufficient condition for dynamic stability. As seen in Fig. 4, given values of k_{33} and k_{11} of 2.0 and 1.3, respectively, when $C_{N_\alpha} < 0.32$, static stability will be accompanied by dynamic instability. That is when equilibrium state oscillations that are disturbed about the increasing amplitude will develop. Figure 4 shows that, for constant α_{11}, an increase in α_{33} increases the frequency of oscillation and reduces the damping rate. A increase in α_{33} is about 30% causes a decrease in damping rate of approximately 80%.

Comparing Figs. 4 and 5 indicates that the main effect of an increase in α_{11} is a reduction of the frequency of oscillation

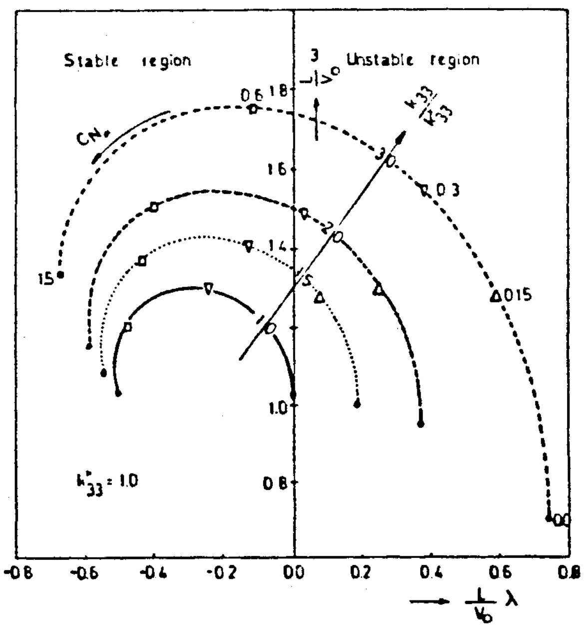

Fig. 4 Root-locus curves showing the effect of α_{33} on dynamic stability.

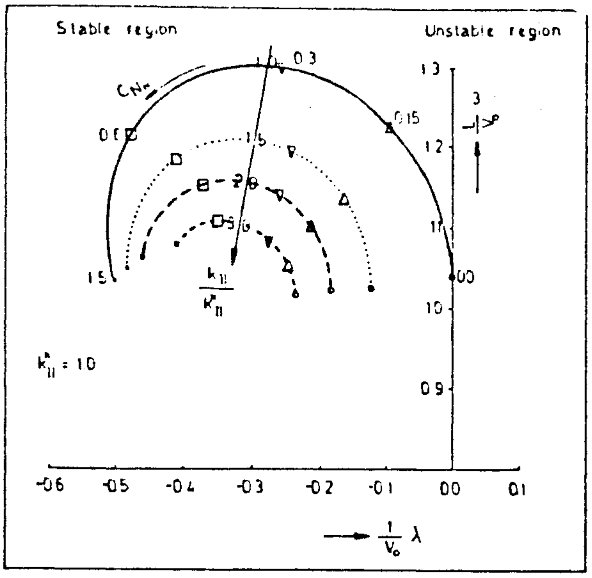

Fig. 5 Root-locus curves showing the effect of α_{11} on dynamic stability.

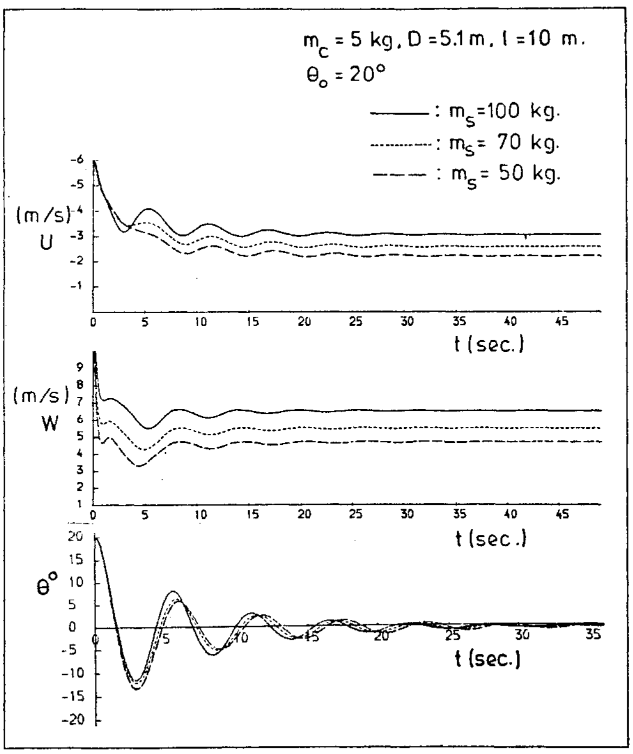

Fig. 6 Variation of pitch angle and descending velocities with time for hemispherical parachute canopies of differing store mass m.

and an increase in the damping rate. Unlike α_{33}, α_{11} has a stabilizing effect on the dynamic stability of the system.

The apparent moment of inertia term α_{55} does not have a considerable influence on stability. The effects of the physical parameters on the dynamic stability are given in detail in Ref. 12.

IV. Prediction of Dynamic Descent Performance

Figure 2 shows that the apparent mass coefficients k_{11} and k_{33} for a parachute store system vary with the canopy shape, the angle of attack, and acceleration numbers δ_x and δ_y. In the early stages of oscillatory motion of the system, the acceleration number varies with time; thus, these experimentally obtained apparent mass components are much larger than values that have been determined by potential flow evaluations.

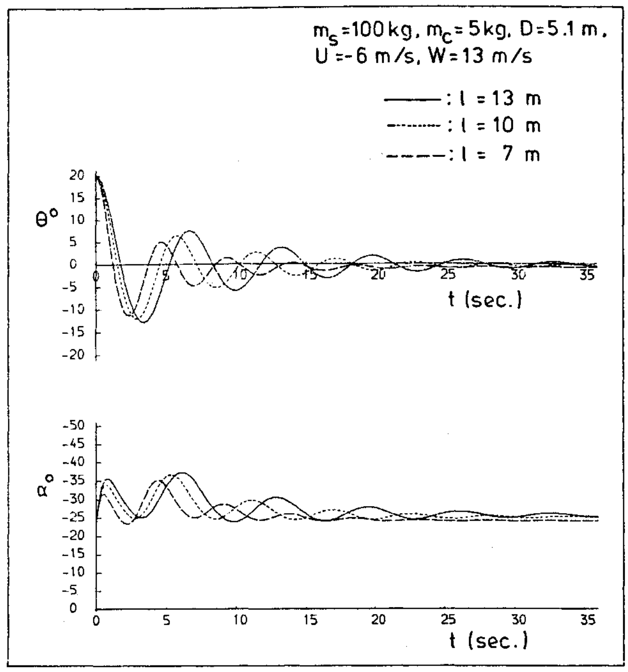

Fig. 7 Variation of pitching angle and angle of attack with time for hemispherical parachute canopies of differing rigging line length L (apparent mass terms are variable, as in Fig. 2).

If the validity of Eqs. (2) is assumed, then, for a parachute store system, linearlized stability analysis indicates that a necessary but insufficient condition for dynamic stability in pitch is that $dC_N/d\alpha$ at equilibrium shall exceed zero. But the dominant cause of varying dynamic response is the magnitude of $dC_N/d\alpha$. Apparent inertia terms and time lag play critical role, in the determination of dynamic descent characteristics for parachutes that have small values of $dC_N/d\alpha$ about the equilibrium angle, such as hemispherical and cruciform (2:1) canopies. As apparent mass terms vary with canopy shape, angle of attack, and acceleration number δ through iterative procedure, these variations can be introduced into the computer model.

The complete equations of motion [Eqs. (2)] for a parachute include terms representative of the size of the parachute canopy, the length of the rigging lines, the mass of the store, and the density of air as well as apparent mass terms. Varying each of these independently in the computer model, their effects on the performance prediction of the system were determined. Results obtained by numerical solution are presented in Figs. 6–10.

If the mass of store is increased, then it is evident that the descending velocity will also increase. Dynamically, Fig. 6 shows that an increase in store mass causes a slight increase in damping rate and oscillation frequency.

Figure 7 indicates that increasing the rigging line length decreases the stability of the system, and this prediction is confirmed by observation. However, an opposite response would be obtained if the constant potential flow values were used for the apparent mass coefficients k_{11} and k_{33}, as shown in Fig. 8.

The effects of canopy size on pitching oscillation become less stable as its damping rate is substantially reduced. Its effect on oscillation frequency is relatively small (Fig. 9). As Knacke[21] pointed out, as the parachute increases in size, stability can only be maintained by reducing canopy porosity. This reduction will directly affect the canopy's steady and unsteady aerodynamic characteristics.

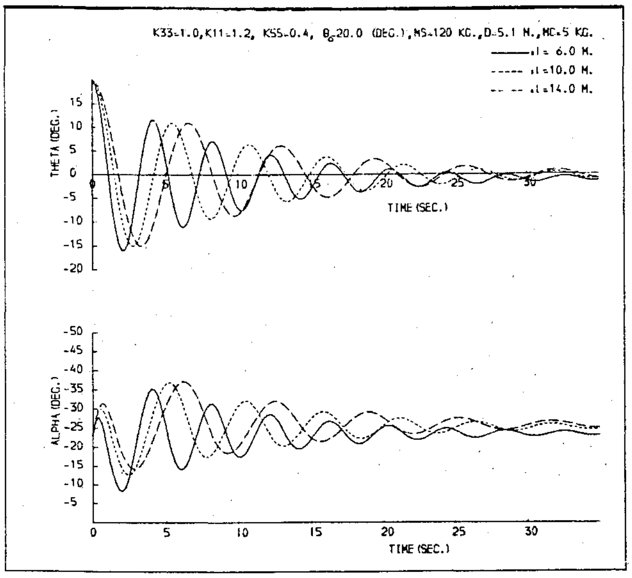

Fig. 8 Variation of pitching angle and angle of attack with time for hemispherical parachute canopies of differing rigging line length L (apparent mass terms are constant).

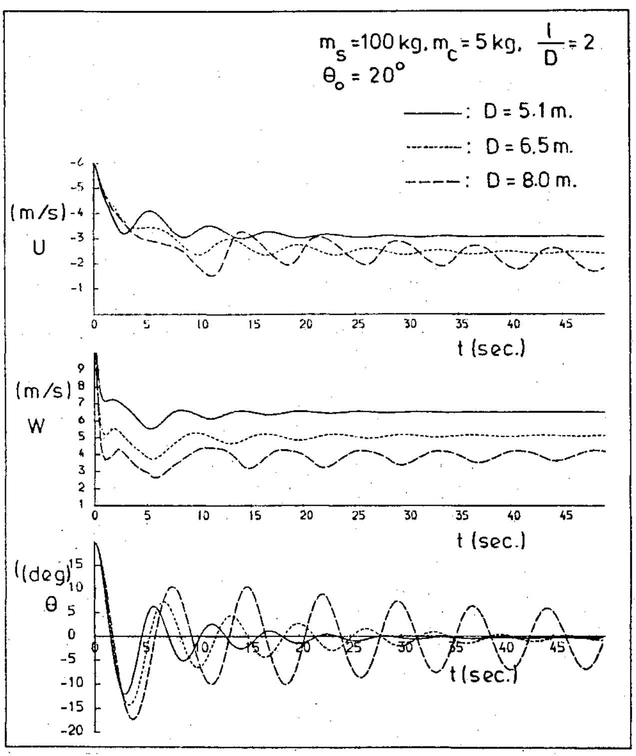

Fig. 9 Variations of pitching angle descending velocities with time for hemispherical parachute canopies of differing diameters D.

At altitude, the system is much more stable then at sea level, as seen in Fig. 10. This is because, as the altitude increases, the steady aerodynamic forces acting on the canopy, $T = \frac{1}{2}\rho A\ C_T V^2$, are reduced due to the decreasing density of air, and hence, the dynamic stability increases.

Inserting the phase lag discussed in Sec. II into the parachute prediction computer model with constant apparent mass terms determined by potential flow evaluation, the effects for hemispherical and cruciform (4:1) canopies are presented in Figs. 11 and 12. As seen in the graphs, the time lag has destabilizing influences on the stability. For parachutes whose values of $dC_N/d\alpha$ about the equilibrium angle are small, the effect of the time lag on the dynamic stability is considerable, such as for a hemispherical canopy (Fig. 11). Increasing time lag reduces the damping rate and period of oscillation. For cruciform parachute canopy with the arm ratio of 4:1, which has high values of $dC_N/d\alpha$, the effect of the time lag is not considerable (Fig. 12).

This result is also obtained when considering the apparent mass effects. Increasing the values of $dC_N/d\alpha$ reduces the effect of the apparent mass terms on the dynamic stability.

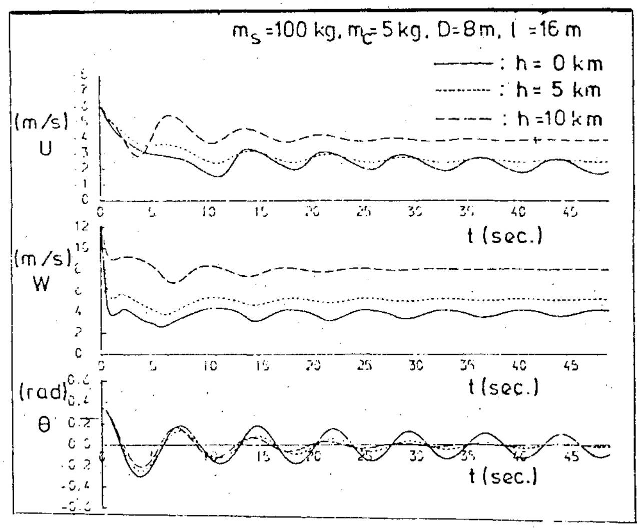

Fig. 10 Variations of performance characteristics with time for hemispherical parachute, the altitude h being varied.

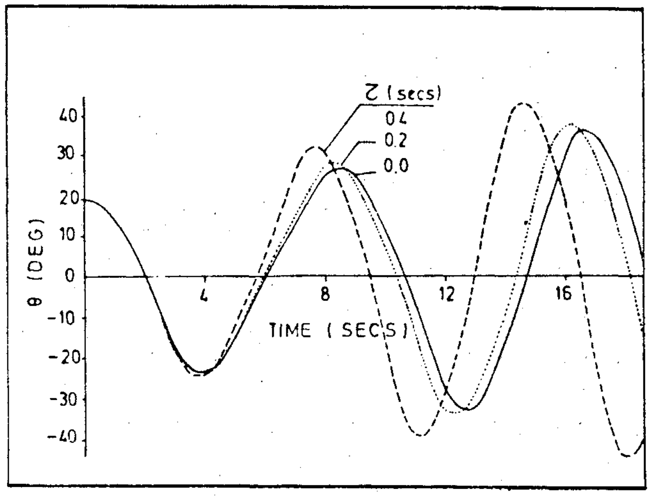

Fig. 11 Variation of pitching angle with phase lag (or time lag) for hemispherical parachute canopy.

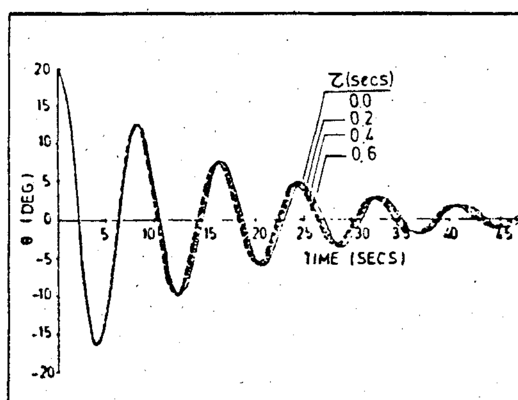

Fig. 12 Variation of pitching angle with phase lag (or time lag) for cruciform (4:1) parachute canopy.

Hence, larger values of $dC_N/d\alpha$, which the cruciform canopy (4:1) has, dominate the dynamic behavior of the system. Consequently, for parachutes with large value of $dC_N/d\alpha$ about the equilibrium angle, there is no need to consider the effects of the apparent mass terms or the time lag on the dynamic stability of the system.

V. Conclusion

Aerodynamic forces and moments developed on unsteadily moving bodies immersed in fluids can differ appreciably in magnitude from those determined in steady motion experiments, depending on the magnitude of the instantaneous dimensionless acceleration numbers. Using experimentally determined apparent mass terms in predicting performance characteristics of the parachute store system gives results that are confirmed by practical observation. Increasing the rigging line length and canopy size and decreasing the altitude have a destabilizing influence on the system oscillation. For parachutes that have high values of $dC_N/d\alpha$ about the equilibrium angle, the apparent mass and the time lag do not play significant roles in determining the performance characteristics of the system.

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